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Closed loop simulation for a magnetic gradiometry mission

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Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

Closed Loop Simulation for a Magnetic Gradiometry Mission

AGU Fall Meeting 2010
GP21A-0986

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Introduction

In the near future the Swarm satellite mission will for the first time measure the East-West gradient of the magnetic field, which contains valuable information on the North-South oriented features of crustal magnetization Going beyond Swarm, we performed a simulation of a full magnetic gradiometry mission, emphasizing on the benefits of measuring the full gradient tensor in addition to the three field components. Using simulated orbits from a low Earth-orbiting satellite, synthetic data of the magnetic field vector and of the nine elements of the magnetic gradient tensor are calculated using a given (input) magnetic field model for the various field contributions (in the core, lithosphere, magnetosphere, and ionosphere). From these synthetic data we estimate field models using either the magnetic vector field measurements only or full gradient observations, and compare our model retrieval with the original (input) model. This study shows qualitatively the scientific benefit of measurements of the gradient tensor in space.

Magnetic Gradiometry

- Measurement of the first derivative of each magnetic field component ($\mathbf{B}_r$, $\mathbf{B}_\theta$, $\mathbf{B}_\varphi$) along each spacial direction ($\mathbf{r}$, $\boldsymbol{\theta}$, $\boldsymbol{\varphi}$)
- In the 3D space that defines the gradient tensor, consisting of 3 × 3 = 9 spatial derivatives and forming a second rank tensor

$$\nabla\mathbf{B} = \begin{pmatrix} \frac{\partial B_r}{\partial r} & \frac{1}{r}\frac{\partial B_r}{\partial \theta} - \frac{1}{r}B_\theta & \frac{1}{r\sin\theta}\frac{\partial B_r}{\partial \phi} - \frac{1}{r}B_\phi \\ \frac{\partial B_\theta}{\partial r} & \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{1}{r}B_r & \frac{1}{r\sin\theta}\frac{\partial B_\theta}{\partial \phi} - \frac{\cot\theta}{r}B_\phi \\ \frac{\partial B_\phi}{\partial r} & \frac{1}{r}\frac{\partial B_\phi}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial \phi} + \frac{1}{r}B_r + \frac{\cot\theta}{r}B_\theta \end{pmatrix} \approx \begin{pmatrix} \frac{\partial B_r}{\partial r} & \frac{1}{r}\frac{\partial B_r}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial B_r}{\partial \phi} \\ \frac{\partial B_\theta}{\partial r} & \frac{1}{r}\frac{\partial B_\theta}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial B_\theta}{\partial \phi} \\ \frac{\partial B_\phi}{\partial r} & \frac{1}{r}\frac{\partial B_\phi}{\partial \theta} & \frac{1}{r\sin\theta}\frac{\partial B_\phi}{\partial \phi} \end{pmatrix}$$

- Approximation when looking for small scale features
- The magnetic field **B** is always a solenoid field: 8 elements instead of 9 to be identified in order to fully determine the gradient tensor

$$\nabla\cdot\mathbf{B} = 0 \Leftrightarrow tr(\nabla\mathbf{B}) = 0$$

- In case we are in a source free environment: 5 instead of 8 elements to be identified in order to fully determine the gradient tensor

$$(\nabla\mathbf{B}) = (\nabla\mathbf{B})^T$$

Magnetic Gradient Tensor Visualization

- Crustal field resulting from Magnetic Field model MF6 for degree values 15≤n≤ 60
- The information contained in the field components is reproduced by the gradients
- Each tensor element enhances certain features of the crustal field
- The gradient tensor provides additional information

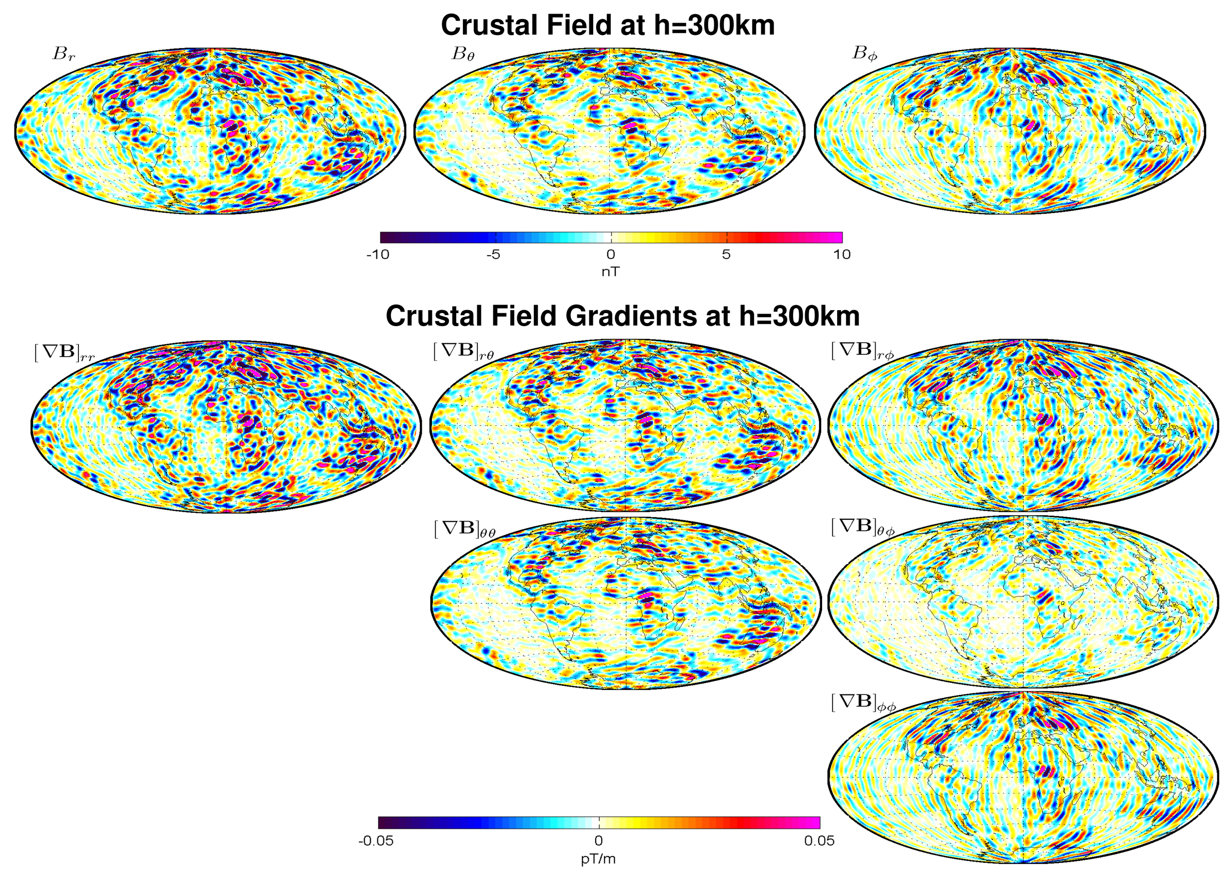

Figure 1: Crustal field (top) and its gradients (bottom) resulting from MF6 model

- $[\nabla \mathrm{B}]_{rr}$: outlines steep boundaries
- $[\nabla \mathrm{B}]_{\theta\theta}$ & $[\nabla \mathrm{B}]_{r\theta}$: outline East-West structures
- $[\nabla \mathrm{B}]_{\varphi\varphi}$ & $[\nabla \mathrm{B}]_{r\varphi}$: outline North-South structures
- $[\nabla \mathrm{B}]_{\theta\varphi}$: outlines body corners

The Simulation

Idea:

- To compare the main field and secular variation recovery using field observations with the recovery using gradient observations

Goal:

- To prove that the measurement of the gradient tensor in space provides higher quality in the recovery of primarily the crustal field.

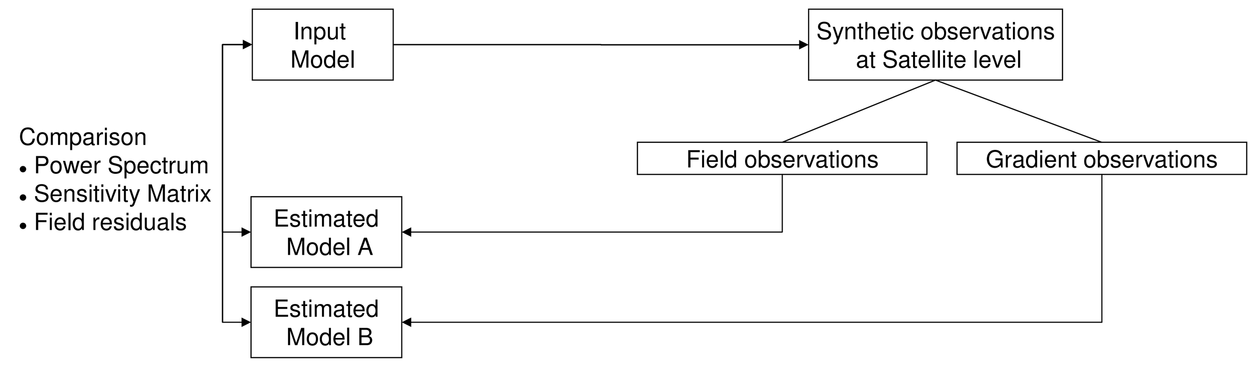

Figure 2: The simulation block diagram

Satellite Orbit:

- Initial Altitude h=450km, inclination i=87.4⁰
- 2 years, 4min sampling rate

Synthetic observations (sum of):

- Static Field (n≤60): Based on CM4 for n≤40 and MF2 for 41≤n≤60
- Secular Variation (n≤19): Cubic splines for n≤10 and linear for 11≤n≤19
- Magnetospheric Field (n≤3): Hour-by-hour spherical harmonic analysis of world wide distributed mean values of the years 1997-2002 in dipole-latitude and magnetic local time
- Ionospheric Field (n≤15; m≤5): Based on CM4 with Sq currents confined to E-region with maximum current density at altitude h=110km

Recovery:

- Static Field for degree values n≤60
- Linear Secular Variation for degree values n≤19
- No Magnetospheric and no Ionospheric field are modeled in the recovery

Results – Power Spectrum and Sensitivity Matrix

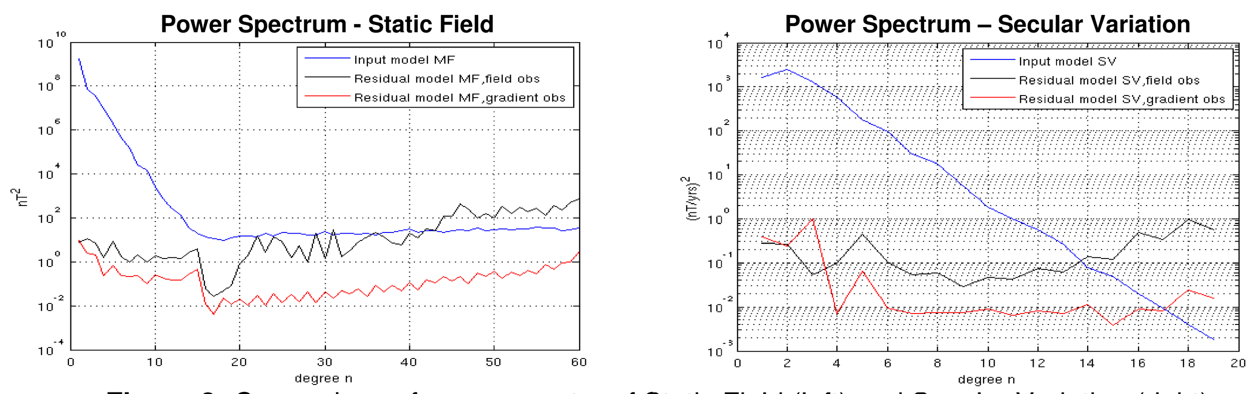

Figure 3: Comparison of power spectra of Static Field (left) and Secular Variation (right)

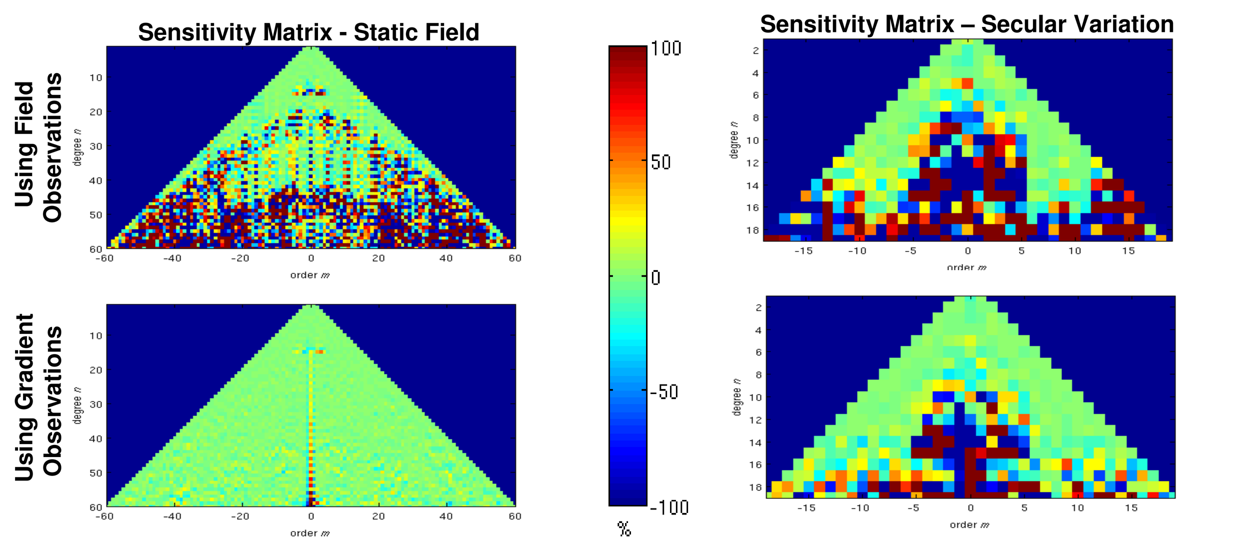

Figure 4: Comparison of sensitivity matrices of Static Field (left) and Secular Variation (right)

- Gradients reconstruct the crustal field with 100 times higher accuracy and the Secular Variation with 10 times higher accuracy than field observations
- Gradients can resolve the crustal field higher than degree 60 and the Secular variation up to degree 17, whereas field observations cannot go higher than degree 40 and 12 respectively
- Gradients suppress the effect of spectral leakage from magnetospheric and ionospheric field which is shown as enhanced diagonal error structures when using field observations
- Gradients recover Secular Variation with higher accuracy compared to field observations especially for tesseral coefficients

Results – Field residuals on ground

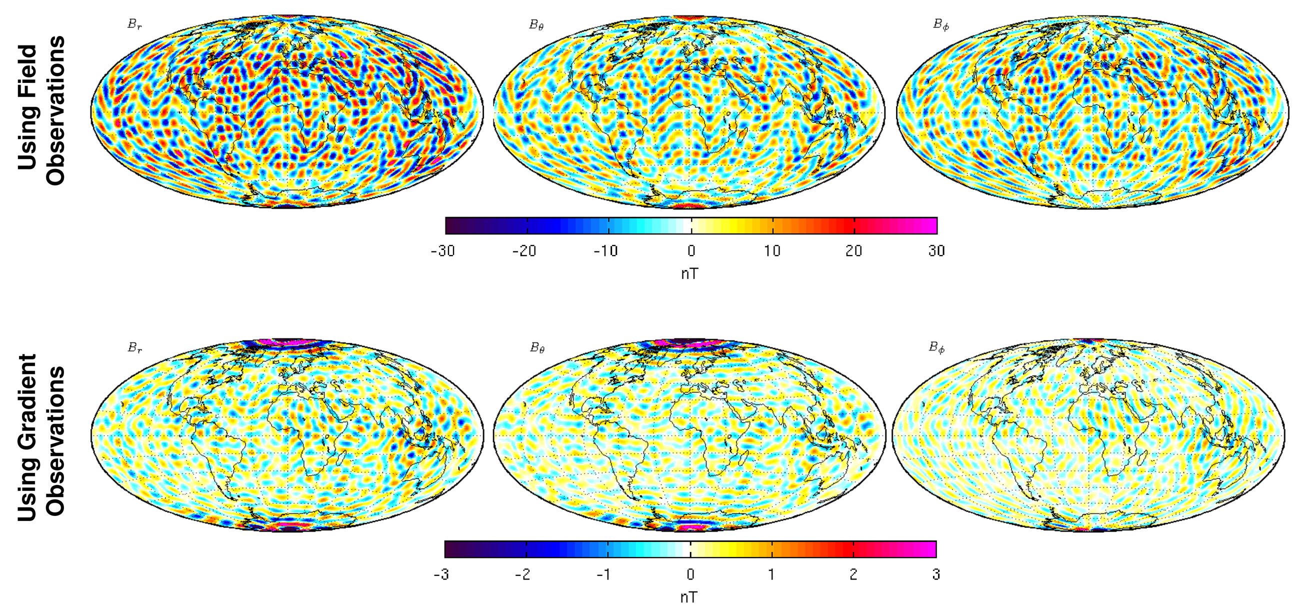

Figure 5: Comparison of field residuals on ground

- Crustal field residuals are calculated for degree 16≤n≤40 since the field observations cannot provide meaningful recovery for degree values n>40
- The field errors on ground are significantly smaller using gradient observations than using field observations
- Gradients show relatively enhanced field errors around the poles due to polar gap. The polar gap effect exists when using field observations also but is hidden by the higher field errors
- The crustal field errors on ground using gradient observations (<±3nT) are not significant with respect to the crustal field range on ground (~±200nT)

Future Preliminary Results

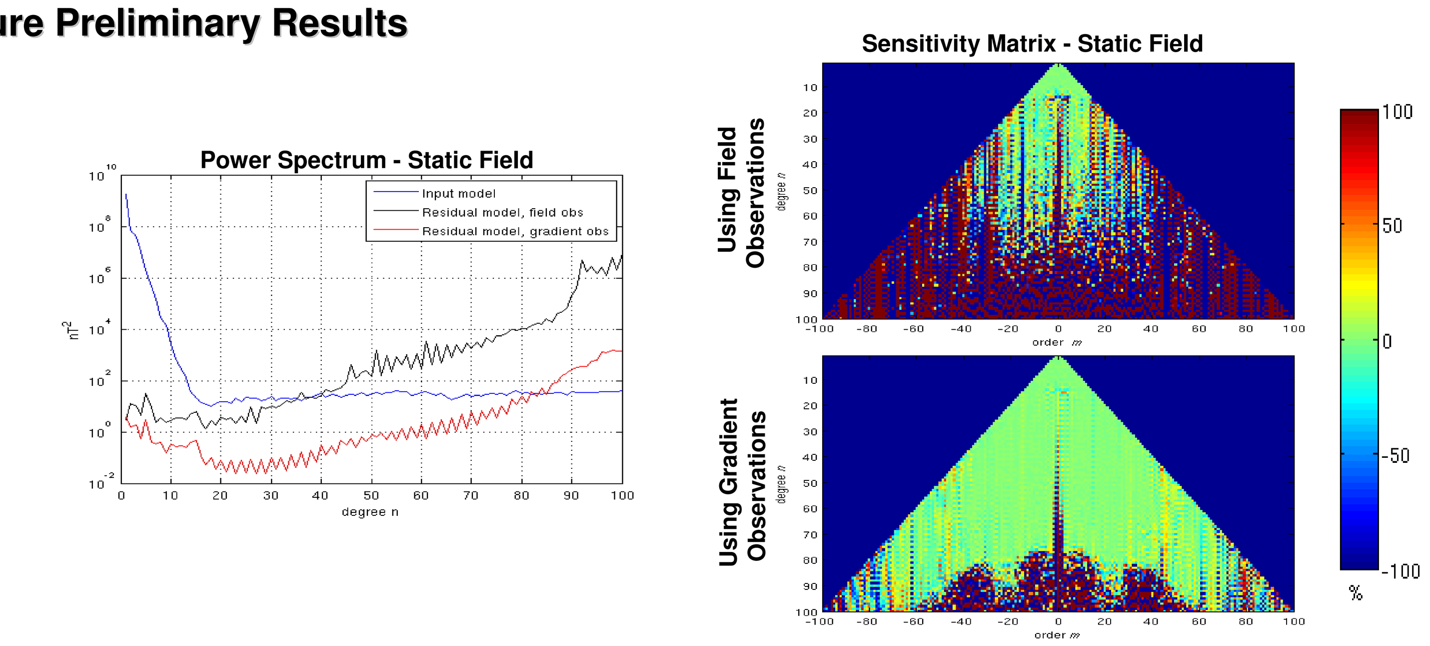

Figure 4: Comparison of power spectra (left) and sensitivity matrices (right)

- Simulation carried out for a higher degree static field n≤150
- For faster computational purposes Secular Variation was neglected and a shorter dataset of 3 months was selected
- Gradients can resolve the crustal field up to degree n=80, whereas field observations cannot resolve it higher than degree n=40
- Spectral leakage appears affecting degree values n>80. The leakage is expected to be significantly reduced using a more dense dataset with 30sec sampling rate

Conclusions

The study shows that the measurement of the magnetic field gradient tensor in space provides significantly better reconstruction of primarily the crustal field, compared to having only measurements of the field components. Moreover, gradient observations not only provide additional information about the crustal field structures but also seem to decorrelate the noise introduced from the highly time dependant field sources such as magnetosphere and ionosphere. Therefore, with gradients we are able to reconstruct the crustal field up to degree n=80 with relatively high accuracy ignoring the magnetospheric and ionospheric field.